RESUMEN DE LAS MATRICES RIGIDEZ DE DISTINTOS TIPOS DE BARRAS

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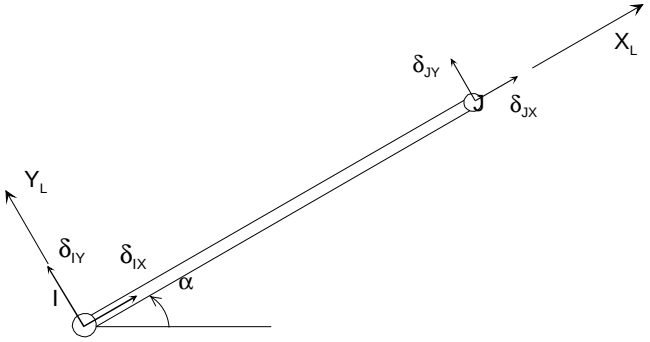
BARRA BIARTICULADA PLANA- RIGIDEZ EN EL SISTEMA LOCAL

$$\begin{Bmatrix} P_{IX} \\ P_{IY} \\ P_{JX} \\ P_{JY} \end{Bmatrix} = \left[\begin{array}{cc|cc} EA/L & 0 & -EA/L & 0 \\ 0 & 0 & 0 & 0 \\ \hline -EA/L & 0 & EA/L & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{Bmatrix} \delta_{IX} \\ \delta_{IY} \\ \delta_{JX} \\ \delta_{JY} \end{Bmatrix}$$

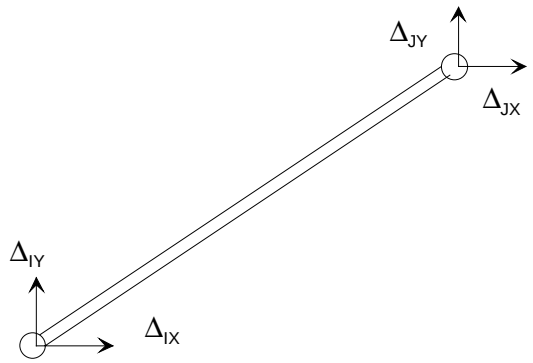


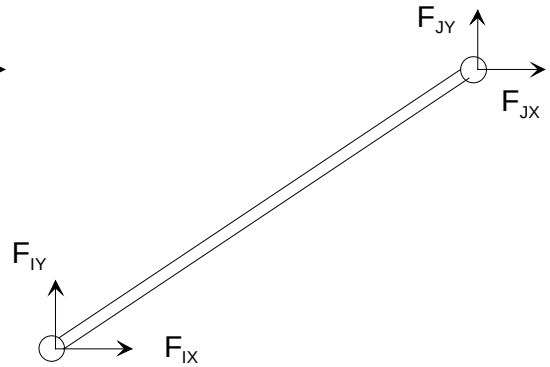
BARRA BIARTICULADA PLANA- RIGIDEZ EN EL SISTEMA GENERAL

$$\begin{Bmatrix} F_{IX} \\ F_{IY} \\ F_{JX} \\ F_{JY} \end{Bmatrix} = \frac{EA}{L} \left[\begin{array}{cc|cc} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ \hline -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{array}\right] \begin{Bmatrix} \Delta_{IX} \\ \Delta_{IY} \\ \Delta_{JX} \\ \Delta_{JY} \end{Bmatrix}$$

s = seno α

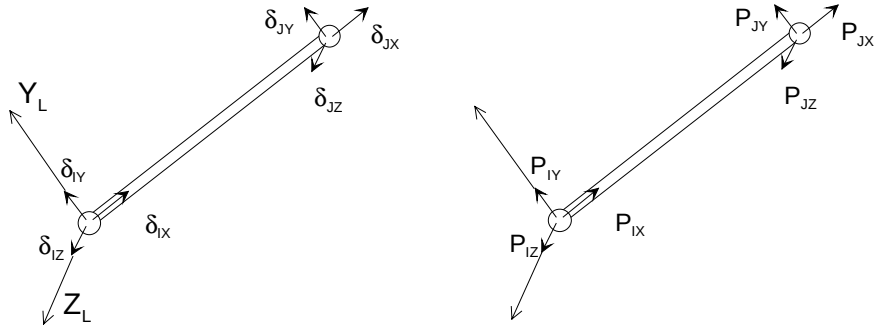
c = coseno α





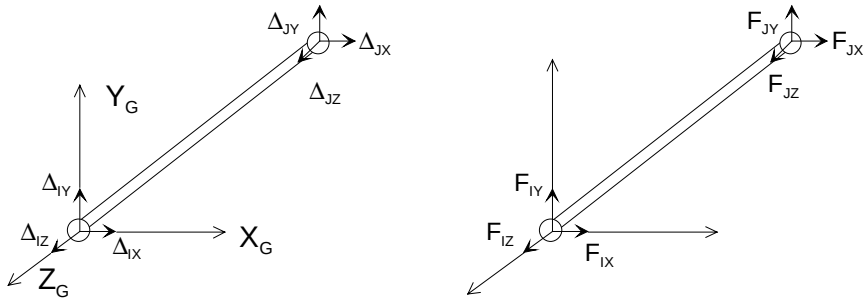
BARRA BIARTICULADA ESPACIAL - RIGIDEZ EN EL SISTEMA LOCAL

$$\begin{Bmatrix} P_{IX} \\ P_{IY} \\ P_{IZ} \\ \hline P_{JX} \\ P_{JY} \\ P_{JZ} \end{Bmatrix} = \frac{EA}{L} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{Bmatrix} \delta_{IX} \\ \delta_{IY} \\ \delta_{IZ} \\ \hline \delta_{JX} \\ \delta_{JY} \\ \delta_{JZ} \end{Bmatrix}$$



BARRA BIARTICULADA ESPACIAL - RIGIDEZ EN EL SISTEMA GENERAL

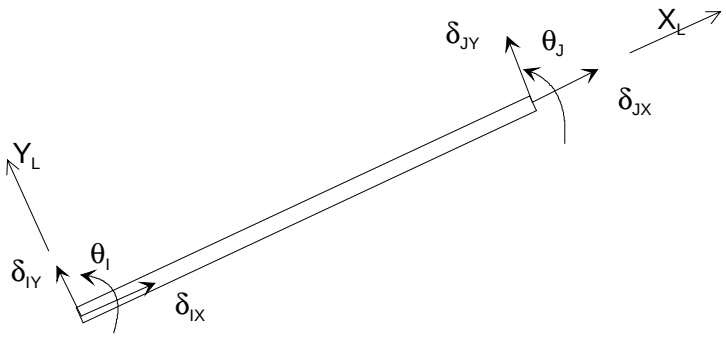
$$\begin{Bmatrix} F_{IX} \\ F_{IY} \\ F_{IZ} \\ \hline F_{JX} \\ F_{JY} \\ F_{JZ} \end{Bmatrix} = EA/L \left[\begin{array}{ccc|ccc} \lambda\lambda & \lambda\mu & \lambda\nu & -\lambda\lambda & -\lambda\mu & -\lambda\nu \\ \mu\lambda & \mu\mu & \mu\nu & -\mu\lambda & -\mu\mu & -\mu\nu \\ \nu\lambda & \nu\mu & \nu\nu & -\nu\lambda & -\nu\mu & -\nu\nu \\ \hline -\lambda\lambda & -\lambda\mu & -\lambda\nu & \lambda\lambda & \lambda\mu & \lambda\nu \\ -\mu\lambda & -\mu\mu & -\mu\nu & \mu\lambda & \mu\mu & \mu\nu \\ -\nu\lambda & -\nu\mu & -\nu\nu & \nu\lambda & \nu\mu & \nu\nu \end{array}\right] \begin{Bmatrix} \Delta_{IX} \\ \Delta_{IY} \\ \Delta_{IZ} \\ \hline \Delta_{JX} \\ \Delta_{JY} \\ \Delta_{JZ} \end{Bmatrix}$$

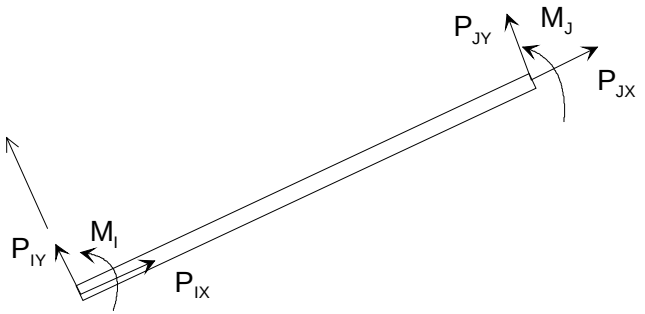


λ = coseno director eje X_L
μ = coseno director eje Y_L
ν = coseno director eje Z_L

VIGA A FLEXIÓN EN EL PLANO - RIGIDEZ EN EL SISTEMA LOCAL

$$\begin{Bmatrix} P_{IX} \\ P_{IY} \\ M_I \\ \hline P_{JX} \\ P_{JY} \\ M_J \end{Bmatrix} = \left[\begin{array}{ccc|ccc} EA/L & 0 & 0 & -EA/L & 0 & 0 \\ 0 & 12EI/L^3 & 6EI/L^2 & 0 & -12EI/L^3 & 6EI/L^2 \\ 0 & 6EI/L^2 & 4EI/L & 0 & -6EI/L^2 & 2EI/L \\ \hline -EA/L & 0 & 0 & EA/L & 0 & 0 \\ 0 & -12EI/L^3 & -6EI/L^2 & 0 & 12EI/L^3 & -6EI/L^2 \\ 0 & 6EI/L^2 & 2EI/L & 0 & -6EI/L^2 & 4EI/L \end{array}\right] \begin{Bmatrix} \delta_{IX} \\ \delta_{IY} \\ \theta_I \\ \hline \delta_{JX} \\ \delta_{JY} \\ \theta_J \end{Bmatrix}$$

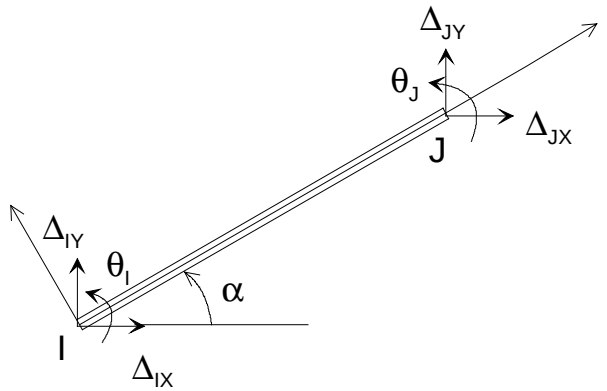




VIGA A FLEXIÓN EN EL PLANO - RIGIDEZ EN EL SISTEMA GENERAL

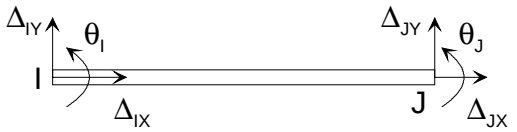
$$\begin{Bmatrix} F_{IX} \\ F_{IY} \\ M_I \\ \hline F_{JX} \\ F_{JY} \\ M_J \end{Bmatrix} = \left[\begin{array}{cccc|cc} EAc^2/L & EAsc/L & -6EIs/L^2 & -EAc^2/L & -EAsc/L & -6EIs/L^2 \\ +12EIs^2/L^3 & -12EIsc/L^3 & & -12EIs^2/L^3 & +12EIsc/L^3 & \\ \hline EAsc/L & EAs^2/L & 6EIc/L^2 & -EAsc/L & -EAs^2/L & 6EIc/L^2 \\ -12EIsc/L^3 & +12EIc^2/L^3 & & +12EIsc/L^3 & -12EIc^2/L^3 & \\ \hline -6EIs/L^2 & 6EIc/L^2 & 4EI/L & 6EIs/L^2 & -6EIc/L^2 & 2EI/L \\ \hline -EAc^2/L & -EAsc/L & 6EIs/L^2 & EAc^2/L & EAsc/L & 6EIs/L^2 \\ -12EIs^2/L^3 & +12EIsc/L^3 & & +12EIs^2/L^3 & -12EIsc/L^3 & \\ \hline -EAsc/L & -EAs^2/L & -6EIc/L^2 & EAsc/L & EAs^2/L & -6EIc/L^2 \\ +12EIsc/L^3 & -12EIc^2/L^3 & & -12EIsc/L^3 & +12EIc^2/L^3 & \\ \hline -6EIs/L^2 & 6EIc/L^2 & 2EI/L & 6EIs/L^2 & -6EIc/L^2 & 4EI/L \end{array}\right] \begin{Bmatrix} \Delta_{IX} \\ \Delta_{IY} \\ \theta_I \\ \hline \Delta_{JX} \\ \Delta_{JY} \\ \theta_J \end{Bmatrix}$$

c = cos α s = sen α



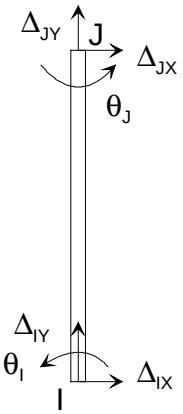
VIGA A FLEXION EN EL PLANO. VIGA HORIZONTAL

$$\begin{Bmatrix} F_{IX} \\ F_{IY} \\ M_I \\ F_{JX} \\ F_{JY} \\ M_J \end{Bmatrix} = \left[\begin{array}{ccc|ccc} EA/L & 0 & 0 & -EA/L & 0 & 0 \\ 0 & 12EI/L^3 & 6EI/L^2 & 0 & -12EI/L^3 & 6EI/L^2 \\ 0 & 6EI/L^2 & 4EI/L & 0 & -6EI/L^2 & 2EI/L \\ \hline -EA/L & 0 & 0 & EA/L & 0 & 0 \\ 0 & -12EI/L^3 & -6EI/L^2 & 0 & 12EI/L^3 & -6EI/L^2 \\ 0 & 6EI/L^2 & 2EI/L & 0 & -6EI/L^2 & 4EI/L \end{array}\right] \begin{Bmatrix} \Delta_{IX} \\ \Delta_{IY} \\ \theta_I \\ \Delta_{JX} \\ \Delta_{JY} \\ \theta_J \end{Bmatrix}$$



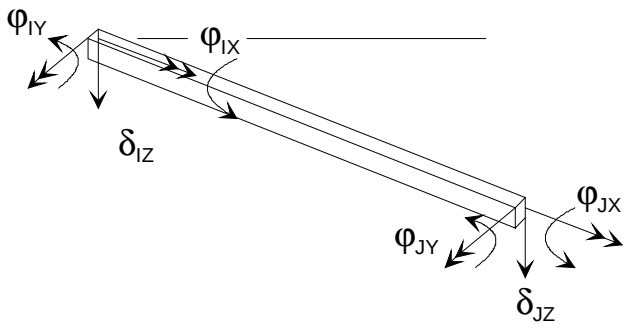
VIGA A FLEXION EN EL PLANO. VIGA VERTICAL

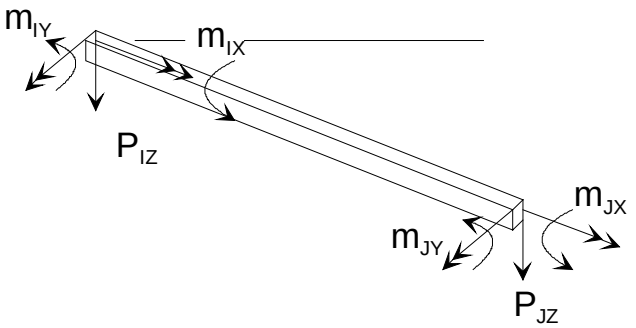
$$\begin{Bmatrix} F_{IX} \\ F_{IY} \\ M_I \\ F_{JX} \\ F_{JY} \\ M_J \end{Bmatrix} = \left[\begin{array}{c|c|c|c|c|c} 12EI/L^3 & 0 & -6EI/L^2 & -12EI/L^3 & 0 & -6EI/L^2 \\ \hline 0 & EA/L & 0 & 0 & -EA/L & 0 \\ \hline -6EI/L^2 & 0 & 4EI/L & 6EI/L^2 & 0 & 2EI/L \\ \hline -12EI/L^3 & 0 & 6EI/L^2 & 12EI/L^3 & 0 & 6EI/L^2 \\ \hline 0 & -EA/L & 0 & 0 & EA/L & 0 \\ \hline -6EI/L^2 & 0 & 2EI/L & 6EI/L^2 & 0 & 4EI/L \end{array}\right] \begin{Bmatrix} \Delta_{IX} \\ \Delta_{IX} \\ \theta_I \\ \Delta_{JX} \\ \Delta_{JY} \\ \theta_J \end{Bmatrix}$$



ELEMENTO DE EMPARRILLADO PLANO - RIGIDEZ EN EL SISTEMA LOCAL

$$\begin{Bmatrix} m_{IX} \\ m_{IY} \\ P_{IZ} \\ \hline m_{JX} \\ m_{JY} \\ P_{JZ} \end{Bmatrix} = \left[\begin{array}{ccc|ccc} GJ/L & 0 & 0 & -GJ/L & 0 & 0 \\ 0 & 4EI_Y/L & -6EI_Y/L^2 & 0 & 2EI_Y/L & 6EI_Y/L^2 \\ 0 & -6EI_Y/L^2 & 12EI_Y/L^3 & 0 & -6EI_Y/L^2 & -12EI_Y/L^3 \\ \hline -GJ/L & 0 & 0 & GJ/L & 0 & 0 \\ 0 & 2EI_Y/L & -6EI_Y/L^2 & 0 & 4EI_Y/L & 6EI_Y/L^2 \\ 0 & 6EI_Y/L^2 & -12EI_Y/L^3 & 0 & 6EI_Y/L^2 & 12EI_Y/L^3 \end{array}\right] \begin{Bmatrix} \varphi_{IX} \\ \varphi_{IY} \\ \delta_{IZ} \\ \hline \varphi_{JX} \\ \varphi_{JY} \\ \delta_{JZ} \end{Bmatrix}$$

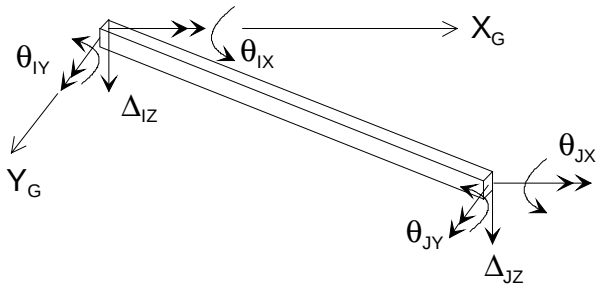


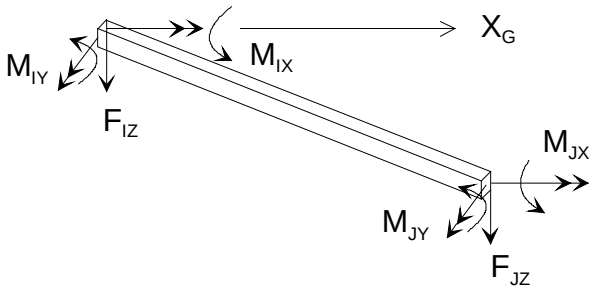


ELEMENTO DE EMPARRILLADO PLANO - RIGIDEZ EN EL SISTEMA GENERAL

$$\begin{Bmatrix} M_{IX} \\ M_{IY} \\ F_{IZ} \\ \hline M_{JX} \\ M_{JY} \\ F_{JZ} \end{Bmatrix} = \left[\begin{array}{cc|c|cc|c} GJc^2/L & GJsc/L & 6EIs/L^2 & -GJc^2/L & -GJsc/L & -6EIs/L^2 \\ +4EIs^2/L & -4EIsc/L & & +2EIs^2/L & -2EIsc/L & \\ \hline GJsc/L & GJs^2/L & -6EIc/L^2 & -GJsc/L & -GJs^2/L & 6EIc/L^2 \\ -4EIsc/L & +4EIc^2/L & & -2EIsc/L & +2EIc^2/L & \\ \hline 6EIs/L^2 & -6EIc/L^2 & 12EI/L^3 & 6EIs/L^2 & -6EIc/L^2 & -12EI/L^3 \\ \hline -GJc^2/L & -GJsc/L & 6EIs/L^2 & GJc^2/L & GJsc/L & -6EIs/L^2 \\ +2EIs^2/L & -2EIsc/L & & +4EIs^2/L & -4EIsc/L & \\ \hline -GJsc/L & -GJs^2/L & -6EIc/L^2 & GJsc/L & GJs^2/L & 6EIc/L^2 \\ -2EIsc/L & +2EIc^2/L & & -4EIsc/L & +4EIc^2/L & \\ \hline -6EIs/L^2 & 6EIc/L^2 & -12EI/L^3 & -6EIs/L^2 & 6EIc/L^2 & 12EI/L^3 \end{array}\right] \begin{Bmatrix} \theta_{IX} \\ \theta_{IY} \\ \Delta_{IZ} \\ \hline \theta_{JX} \\ \theta_{JY} \\ \Delta_{JZ} \end{Bmatrix}$$

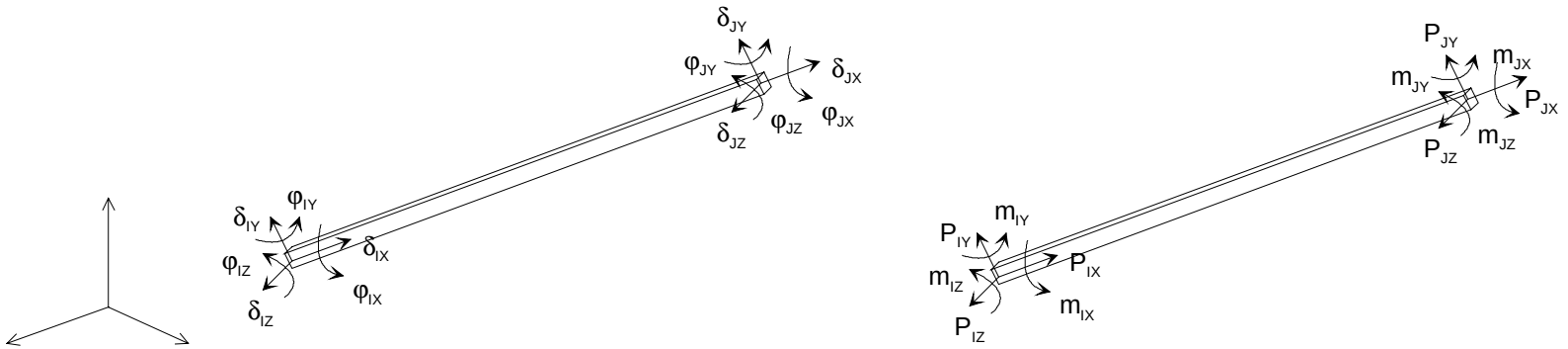
$c = \cos(\alpha)$ $s = sen(\alpha)$ y $I \equiv I_Y$



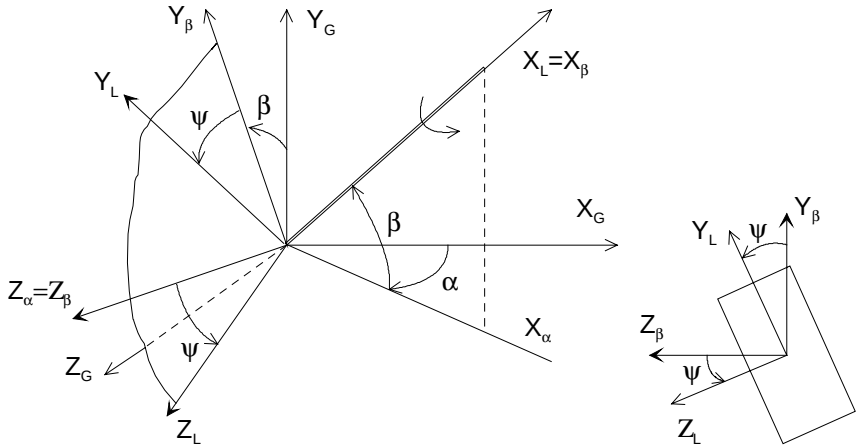


ELEMENTO VIGA ESPACIAL - RIGIDEZ EN EL SISTEMA LOCAL

$$
\begin{Bmatrix} P_{IX} \\ P_{IY} \\ P_{IZ} \\ m_{IX} \\ m_{IY} \\ m_{IZ} \\ \hline P_{JX} \\ P_{JY} \\ P_{JZ} \\ m_{JX} \\ m_{JY} \\ m_{JZ} \end{Bmatrix} =
\left[\begin{array}{cccccc|cccccc}
EA/L & 0 & 0 & 0 & 0 & 0 & -EA/L & 0 & 0 & 0 & 0 & 0 \\
0 & 12I_z/L^3 & 0 & 0 & 0 & 6I_z/L^2 & 0 & -12I_z/L^3 & 0 & 0 & 0 & 6I_z/L^2 \\
0 & 0 & 12I_Y/L^3 & 0 & -6I_Y/L^2 & 0 & 0 & 0 & -12I_Y/L^3 & 0 & -6I_Y/L^2 & 0 \\
0 & 0 & 0 & GJ/L & 0 & 0 & 0 & 0 & 0 & -GJ/L & 0 & 0 \\
0 & 0 & -6I_Y/L^2 & 0 & 4I_Y/L & 0 & 0 & 0 & 6I_Y/L^2 & 0 & 2I_Y/L & 0 \\
0 & 6I_z/L^2 & 0 & 0 & 0 & 4I_z/L & 0 & -6I_z/L^2 & 0 & 0 & 0 & 2I_z/L \\
\hline
-EA/L & 0 & 0 & 0 & 0 & 0 & EA/L & 0 & 0 & 0 & 0 & 0 \\
0 & -12I_z/L^3 & 0 & 0 & 0 & -6I_z/L^2 & 0 & 12I_z/L^3 & 0 & 0 & 0 & -6I_z/L^2 \\
0 & 0 & -12I_Y/L^3 & 0 & 6I_Y/L^2 & 0 & 0 & 0 & 12I_Y/L^3 & 0 & 6I_Y/L^2 & 0 \\
0 & 0 & 0 & -GJ/L & 0 & 0 & 0 & 0 & 0 & GJ/L & 0 & 0 \\
0 & 0 & -6I_Y/L^2 & 0 & 2I_Y/L & 0 & 0 & 0 & 6I_Y/L^2 & 0 & 4I_Y/L & 0 \\
0 & 6I_z/L^2 & 0 & 0 & 0 & 2I_z/L & 0 & -6I_z/L^2 & 0 & 0 & 0 & 4I_z/L
\end{array}\right]
\begin{Bmatrix} \delta_{IX} \\ \delta_{IY} \\ \delta_{IZ} \\ \varphi_{IX} \\ \varphi_{IY} \\ \varphi_{IZ} \\ \hline \delta_{JX} \\ \delta_{JY} \\ \delta_{JZ} \\ \varphi_{JX} \\ \varphi_{JY} \\ \varphi_{JZ} \end{Bmatrix}
$$

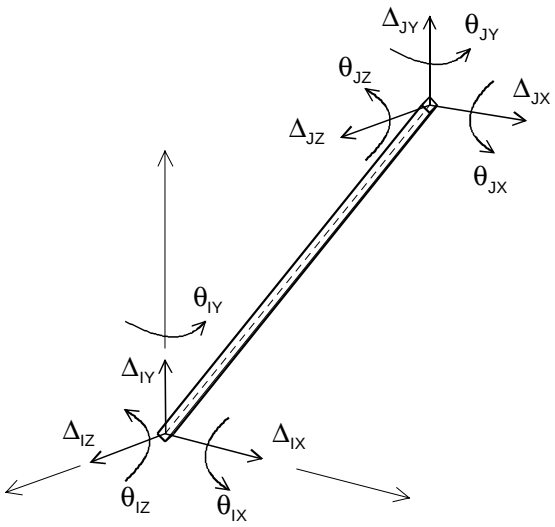


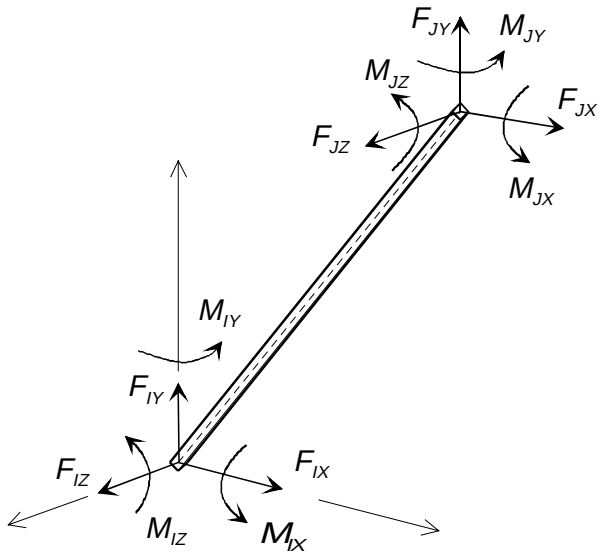
ELEMENTO VIGA ESPACIAL- MATRIZ DE ROTACIÓN



$$\mathbf{T} = \begin{bmatrix} \lambda & \mu & \nu \\ (-\lambda\mu\cos\psi - \nu sen\psi)/D & D\cos\psi & (-\mu\nu\cos\psi + \lambda sen\psi)/D \\ (\lambda\mu sen\psi - \nu\cos\psi)/D & -Dsen\psi & (\mu\nu sen\psi + \lambda\cos\psi)/D \end{bmatrix}$$

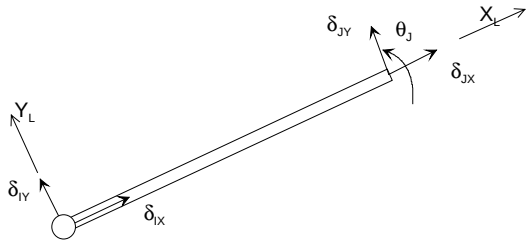
$$D^2 = \lambda^2 + \nu^2$$

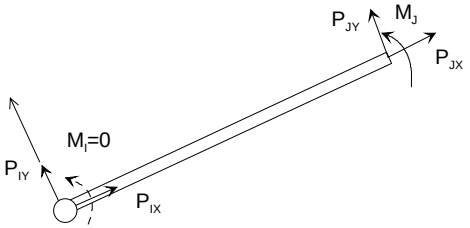




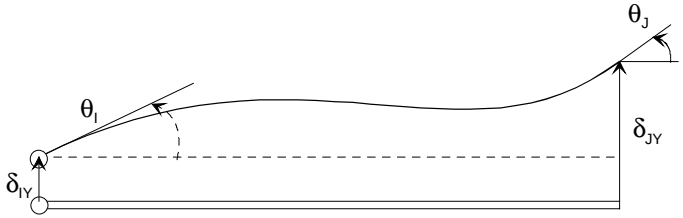
ELEMENTO ARTICULADO - EMPOTRADO - RIGIDEZ EN EL SISTEMA LOCAL

$$\begin{Bmatrix} P_{IX} \\ P_{IY} \\ M_I \\ P_{JX} \\ P_{JY} \\ M_J \end{Bmatrix} = \left[\begin{array}{ccc|ccc} EA/L & 0 & 0 & -EA/L & 0 & 0 \\ 0 & 3EI/L^3 & 0 & 0 & -3EI/L^3 & 3EI/L^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline -EA/L & 0 & 0 & EA/L & 0 & 0 \\ 0 & -3EI/L^3 & 0 & 0 & 3EI/L^3 & -3EI/L^2 \\ 0 & 3EI/L^2 & 0 & 0 & -3EI/L^2 & 3EI/L \end{array}\right] \begin{Bmatrix} \delta_{IX} \\ \delta_{IY} \\ \theta_I \\ \delta_{JX} \\ \delta_{JY} \\ \theta_J \end{Bmatrix}$$



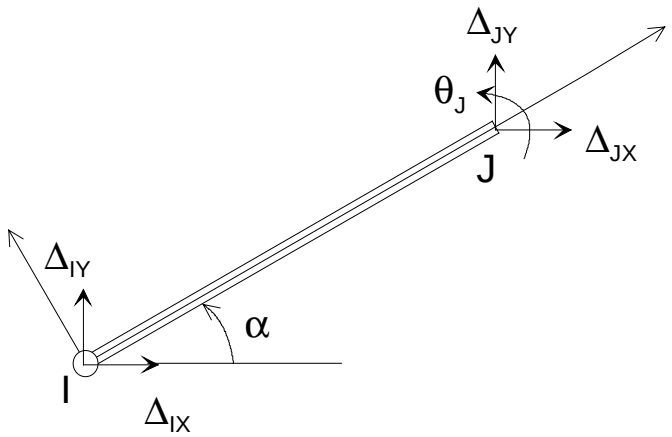


Giro en la articulación $\theta_I = \frac{3}{2L}(\delta_{JY} - \delta_{IY}) - \frac{\theta_J}{2}$



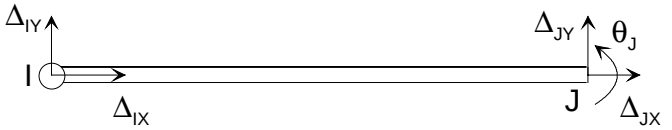
ELEMENTO ARTICULADO - EMPOTRADO - RIGIDEZ EN EL SISTEMA GENERAL

$$\begin{Bmatrix} F_{IX} \\ F_{IY} \\ M_I \\ \hline F_{JX} \\ F_{JY} \\ M_J \end{Bmatrix} = \left[\begin{array}{cc|c|cc|c} EAc^2/L & EAsc/L & 0 & -EAc^2/L & -EAsc/L & -3EIs/L^2 \\ +3EIs^2/L^3 & -3EIsc/L^3 & & -3EIs^2/L^3 & +3EIsc/L^3 & \\ \hline EAsc/L & EAs^2/L & 0 & -EAsc/L & -EAs^2/L & 3EIc/L^2 \\ -3EIsc/L^3 & +3EIc^2/L^3 & & +3EIsc/L^3 & -3EIc^2/L^3 & \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ \hline -EAc^2/L & -EAsc/L & 0 & EAc^2/L & EAsc/L & 3EIs/L^2 \\ -3EIs^2/L^3 & +3EIsc/L^3 & & +3EIs^2/L^3 & -3EIsc/L^3 & \\ \hline -EAsc/L & -EAs^2/L & 0 & EAsc/L & EAs^2/L & -3EIc/L^2 \\ +3EIsc/L^3 & -3EIc^2/L^3 & & -3EIsc/L^3 & +3EIc^2/L^3 & \\ \hline -3EIs/L^2 & 3EIc/L^2 & 0 & 3EIs/L^2 & -3EIc/L^2 & 3EI/L \end{array}\right] \begin{Bmatrix} \Delta_{IX} \\ \Delta_{IX} \\ \theta_I \\ \hline \Delta_{JX} \\ \Delta_{JY} \\ \theta_J \end{Bmatrix}$$



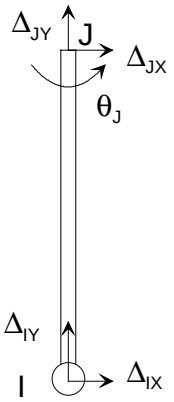
ELEMENTO ARTICULADO - EMPOTRADO HORIZONTAL - RIGIDEZ EN EL SISTEMA GENERAL

$$\begin{Bmatrix} F_{IX} \\ F_{IY} \\ M_I \\ \hline F_{JX} \\ F_{JY} \\ M_J \end{Bmatrix} = \left[\begin{array}{ccc|ccc} EA/L & 0 & 0 & -EA/L & 0 & 0 \\ 0 & 3EI/L^3 & 0 & 0 & -3EI/L^3 & 3EI/L^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline -EA/L & 0 & 0 & EA/L & 0 & 0 \\ 0 & -3EI/L^3 & 0 & 0 & 3EI/L^3 & -3EI/L^2 \\ 0 & 3EI/L^2 & 0 & 0 & -3EI/L^2 & 3EI/L \end{array}\right] \begin{Bmatrix} \Delta_{IX} \\ \Delta_{IY} \\ \theta_I \\ \hline \Delta_{JX} \\ \Delta_{JY} \\ \theta_J \end{Bmatrix}$$



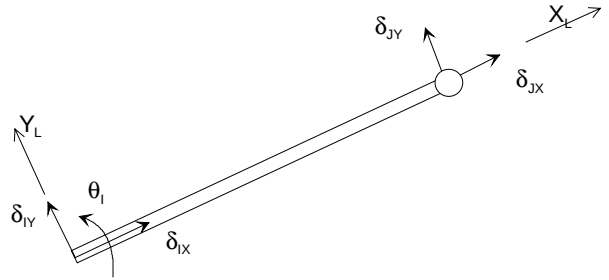
ELEMENTO ARTICULADO - EMPOTRADO VERTICAL- RIGIDEZ EN EL SISTEMA GENERAL

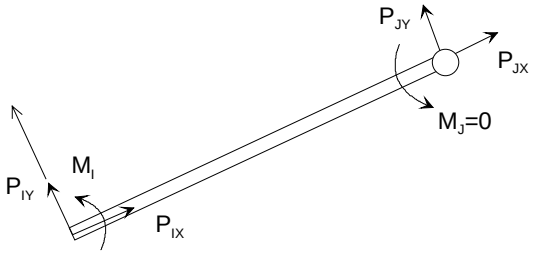
$$\begin{Bmatrix} F_{IX} \\ F_{IY} \\ M_I \\ \hline F_{JX} \\ F_{JY} \\ M_J \end{Bmatrix} = \left[\begin{array}{ccc|ccc} 3EI/L^3 & 0 & 0 & -3EI/L^3 & 0 & -3EI/L^2 \\ \hline 0 & EA/L & 0 & 0 & -EA/L & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ \hline -3EI/L^3 & 0 & 0 & 3EI/L^3 & 0 & 3EI/L^2 \\ \hline 0 & -EA/L & 0 & 0 & EA/L & 0 \\ \hline -3EI/L^2 & 0 & 0 & 3EI/L^2 & 0 & 3EI/L \end{array}\right] \begin{Bmatrix} \Delta_{IX} \\ \Delta_{IX} \\ \theta_I \\ \hline \Delta_{JX} \\ \Delta_{JY} \\ \theta_J \end{Bmatrix}$$



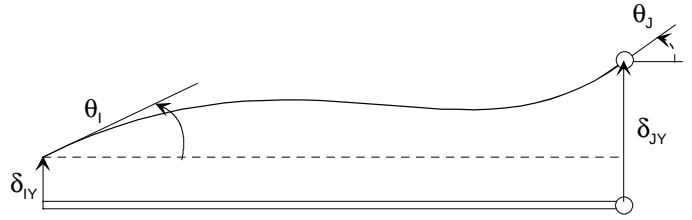
ELEMENTO EMPOTRADO - ARTICULADO - RIGIDEZ EN EL SISTEMA LOCAL

$$\begin{Bmatrix} P_{IX} \\ P_{IY} \\ M_I \\ \hline P_{JX} \\ P_{JY} \\ M_J \end{Bmatrix} = \left[\begin{array}{ccc|ccc} EA/L & 0 & 0 & -EA/L & 0 & 0 \\ 0 & 3EI/L^3 & 3EI/L^2 & 0 & -3EI/L^3 & 0 \\ 0 & 3EI/L^2 & 3EI/L & 0 & -3EI/L^2 & 0 \\ \hline -EA/L & 0 & 0 & EA/L & 0 & 0 \\ 0 & -3EI/L^3 & -3EI/L^2 & 0 & 3EI/L^3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{Bmatrix} \delta_{IX} \\ \delta_{IY} \\ \theta_I \\ \hline \delta_{JX} \\ \delta_{JY} \\ \theta_J \end{Bmatrix}$$



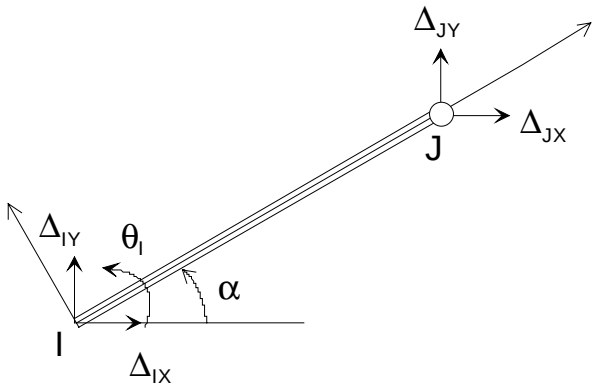


Giro en la articulación $\theta_J = \frac{3}{2L}(\delta_{JY} - \delta_{IY}) - \frac{\theta_I}{2}$



ELEMENTO EMPOTRADO - ARTICULADO - RIGIDEZ EN EL SISTEMA GENERAL

$$\begin{Bmatrix} F_{IX} \\ F_{IY} \\ M_I \\ F_{JX} \\ F_{JY} \\ M_J \end{Bmatrix} = \left[\begin{array}{c|c|c|c|c|c} EAc^2/L & EAsc/L & -3EIs/L^2 & -EAc^2/L & -EAsc/L & 0 \\ +3EIs^2/L^3 & -3EIsc/L^3 & & -3EIs^2/L^3 & +3EIsc/L^3 & \\ \hline EAsc/L & EAs^2/L & 3EIc/L^2 & -EAsc/L & -EAs^2/L & 0 \\ -3EIsc/L^3 & +3EIc^2/L^3 & & +3EIsc/L^3 & -3EIc^2/L^3 & \\ \hline -3EIs/L^2 & 3EIc/L^2 & 3EI/L & 3EIs/L^2 & -3EIc/L^2 & 0 \\ \hline -EAc^2/L & -EAsc/L & 3EIs/L^2 & EAc^2/L & EAsc/L & 0 \\ -3EIs^2/L^3 & +3EIsc/L^3 & & +3EIs^2/L^3 & -3EIsc/L^3 & \\ \hline -EAsc/L & -EAs^2/L & -3EIc/L^2 & EAsc/L & EAs^2/L & 0 \\ +3EIsc/L^3 & -3EIc^2/L^3 & & -3EIsc/L^3 & +3EIc^2/L^3 & \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{Bmatrix} \Delta_{IX} \\ \Delta_{IX} \\ \theta_I \\ \Delta_{JX} \\ \Delta_{JY} \\ \theta_J \end{Bmatrix}$$

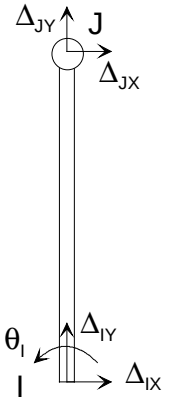


ELEMENTO EMPOTRADO - ARTICULADO HORIZONTAL - RIGIDEZ EN EL SISTEMA GENERAL

$$\begin{Bmatrix} F_{IX} \\ F_{IY} \\ M_I \\ \hline F_{JX} \\ F_{JY} \\ M_J \end{Bmatrix} = \left[\begin{array}{ccc|ccc} EA/L & 0 & 0 & -EA/L & 0 & 0 \\ 0 & 3EI/L^3 & 3EI/L^2 & 0 & -3EI/L^3 & 0 \\ 0 & 3EI/L^2 & 3EI/L & 0 & -3EI/L^2 & 0 \\ \hline -EA/L & 0 & 0 & EA/L & 0 & 0 \\ 0 & -3EI/L^3 & -3EI/L^2 & 0 & 3EI/L^3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{Bmatrix} \Delta_{IX} \\ \Delta_{IY} \\ \theta_I \\ \hline \Delta_{JX} \\ \Delta_{JY} \\ \theta_J \end{Bmatrix}$$

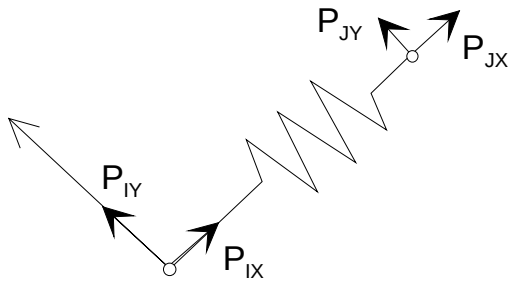
ELEMENTO EMPOTRADO - ARTICULADO VERTICAL - RIGIDEZ EN EL SISTEMA GENERAL

$$\begin{Bmatrix} F_{IX} \\ F_{IY} \\ M_I \\ \hline F_{JX} \\ F_{JY} \\ M_J \end{Bmatrix} = \left[\begin{array}{c|c|c|c|c|c} 3EI/L^3 & 0 & -3EI/L^2 & -3EI/L^3 & 0 & 0 \\ \hline 0 & EA/L & 0 & 0 & -EA/L & 0 \\ \hline -3EI/L^2 & 0 & 3EI/L & 3EI/L^2 & 0 & 0 \\ \hline -3EI/L^3 & 0 & 3EI/L^2 & 3EI/L^3 & 0 & 0 \\ \hline 0 & -EA/L & 0 & 0 & EA/L & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{Bmatrix} \Delta_{IX} \\ \Delta_{IX} \\ \theta_I \\ \hline \Delta_{JX} \\ \Delta_{JY} \\ \theta_J \end{Bmatrix}$$



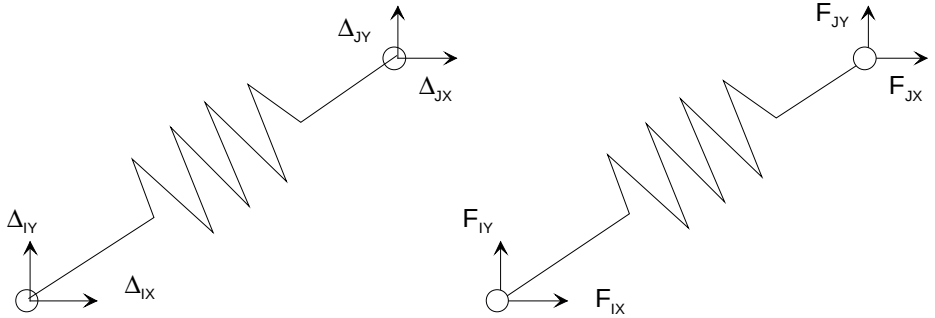
MUELLES DE ESFUERZO AXIAL - RIGIDEZ EN EL SISTEMA LOCAL

$$\begin{Bmatrix} P_{IX} \\ P_{IY} \\ \hline P_{JX} \\ P_{JY} \end{Bmatrix} = \left[\begin{array}{cc|cc} K & 0 & -K & 0 \\ 0 & 0 & 0 & 0 \\ \hline -K & 0 & K & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{Bmatrix} \delta_{IX} \\ \delta_{IY} \\ \hline \delta_{JX} \\ \delta_{JY} \end{Bmatrix}$$



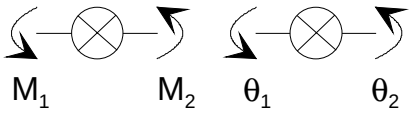
MUELLES DE ESFUERZO AXIAL - RIGIDEZ EN EL SISTEMA LOCAL

$$\begin{Bmatrix} F_{IX} \\ F_{IY} \\ \hline F_{JX} \\ F_{JY} \end{Bmatrix} = K \left[\begin{array}{cc|cc} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ \hline -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{array}\right] \begin{Bmatrix} \Delta_{IX} \\ \Delta_{IY} \\ \hline \Delta_{JX} \\ \Delta_{JY} \end{Bmatrix}$$



MUELLES AL GIRO

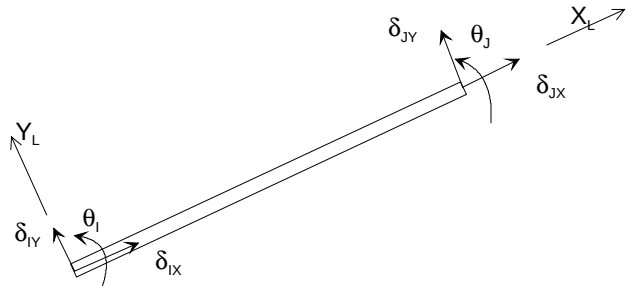
$$\begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix} = \begin{bmatrix} K & -K \\ -K & K \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix}$$

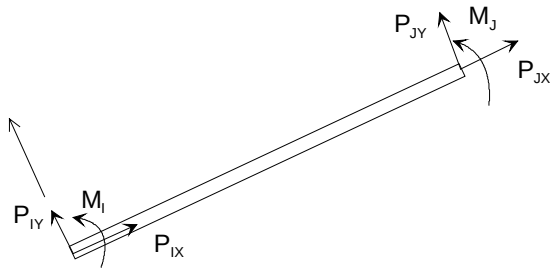


ELEMENTO VIGA PLANA CON ENERGÍA DE ESFUERZO CORTANTE - RIGIDEZ EN EL SISTEMA LOCAL

$$\begin{Bmatrix} P_{IX} \\ P_{IY} \\ M_I \\ P_{JX} \\ P_{JY} \\ M_J \end{Bmatrix} = \left[\begin{array}{ccc|ccc} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{(1+\kappa)L^3} & \frac{6EI}{(1+\kappa)L^2} & 0 & -\frac{12EI}{(1+\kappa)L^3} & \frac{6EI}{(1+\kappa)L^2} \\ 0 & \frac{6EI}{(1+\kappa)L^2} & \frac{(4+\kappa)EI}{(1+\kappa)L} & 0 & -\frac{6EI}{(1+\kappa)L^2} & \frac{(2-\kappa)EI}{(1+\kappa)L} \\ \hline -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{(1+\kappa)L^3} & -\frac{6EI}{(1+\kappa)L^2} & 0 & \frac{12EI}{(1+\kappa)L^3} & -\frac{6EI}{(1+\kappa)L^2} \\ 0 & \frac{6EI}{(1+\kappa)L^2} & \frac{(2-\kappa)EI}{(1+\kappa)L} & 0 & -\frac{6EI}{(1+\kappa)L^2} & \frac{(4+\kappa)EI}{(1+\kappa)L} \end{array}\right] \begin{Bmatrix} \delta_{IX} \\ \delta_{IY} \\ \theta_I \\ \delta_{JX} \\ \delta_{JY} \\ \theta_J \end{Bmatrix}$$

$$\kappa = \frac{12EI}{GA' L^2}$$





ELEMENTO DE EMPARRILLADO PLANO CON ENERGÍA DE ESFUERZO CORTANTE – RIGIDEZ EN EL SISTEMA LOCAL

$$\begin{Bmatrix} m_{IX} \\ m_{IY} \\ P_{IZ} \\ m_{JX} \\ m_{JY} \\ P_{JZ} \end{Bmatrix} = \left[\begin{array}{ccc|ccc} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{(4+\kappa)EI_Y}{(1+\kappa)L} & -\frac{6EI_Y}{(1+\kappa)L^2} & 0 & \frac{(2-\kappa)EI_Y}{(1+\kappa)L} & \frac{6EI_Y}{(1+\kappa)L^2} \\ 0 & -\frac{6EI_Y}{(1+\kappa)L^2} & \frac{12EI_Y}{(1+\kappa)L^3} & 0 & -\frac{6EI_Y}{(1+\kappa)L^2} & -\frac{12EI_Y}{(1+\kappa)L^3} \\ \hline -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & \frac{(2-\kappa)EI_Y}{(1+\kappa)L} & -\frac{6EI_Y}{(1+\kappa)L^2} & 0 & \frac{(4+\kappa)EI_Y}{(1+\kappa)L} & \frac{6EI_Y}{(1+\kappa)L^2} \\ 0 & \frac{6EI_Y}{(1+\kappa)L^2} & -\frac{12EI_Y}{(1+\kappa)L^3} & 0 & \frac{6EI_Y}{(1+\kappa)L^2} & \frac{12EI_Y}{(1+\kappa)L^3} \end{array}\right] \begin{Bmatrix} \varphi_{IX} \\ \varphi_{IY} \\ \delta_{IZ} \\ \varphi_{JX} \\ \varphi_{JY} \\ \delta_{JZ} \end{Bmatrix}$$

$$\kappa = \frac{12EI_Y}{GA' L^2}$$